# Two-Dimensional Simulation of the Angle of Repose for a Particle System with Electrostatic Charge under Lunar and Earth Gravity

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**Abstract:** This paper presents a study of the angle of repose of a two-dimensional particle system under the Earth and Moon gravity fields. The particles interact with electrostatic forces in addition to friction. A two-dimensional discrete element method is used in this analysis with two particle shapes, circular and noncircular. The noncircular shape is constructed with overlapping pairs of disks. For the range of parameters studied, the angle of repose shows little sensitivity to gravity. The sensitivity to friction and electrostatic charges can be either significant or negligible, depending on the range of these values. For each contact friction, there is a threshold of electric charge on the particle such that the angle of repose suddenly drops to zero when the charge exceeds this threshold. The existence of this threshold, once validated in three-dimensional systems, may provide an opportunity to measure the electrostatic charges of the lunar dust in situ.

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## Introduction

During the Apollo missions it was discovered that electrically charged dust on the lunar surface created operational problems. These dust particles adhered to the space suit of the astronauts and were transported back to the space vehicle. After liftoff, these dust particles floated in the cabin, and presented breathing hazards in addition to visual obstruction (Heiken et al. 1991). The dust particles have a broad range of size distributions with the medium around 100 μm. These particles have extremely low electrical conductivity and dielectric losses. The combination of these properties allows them to be easily charged and to maintain their charge. There are three known mechanisms to charge these particles: photoemission, interaction with a photoelectric sheath, and triboelectric charge (Sickafoose et al. 2001). The first of these three is the focus of this study. It applies to the surface layer of lunar particles, which concerns a majority of in-situ operations.

Photoemission from UV light of the Sun charges the particles lying on the surface of the Moon. It induces particle motion across the lunar terminator due to the drastic change of UV input between lunar sunrise and sunset (De and Criswell 1977). The UV input also strongly affects the electrical conductivity of these dust particles (Olhoeft et al. 1973). The reduction of conductivity during lunar night makes charged dust particle more likely to coat space suits and any other exposed surfaces.

The level of photoemission-induced electric charge is not known in situ, although laboratory tests that simulated the lunar condition using various pure mineral particles and lunar and Mars simulants have shown a medium charge at $10^{-14}$ C (Sickafoose et al. 2001; Sternovsky and Robertson 2002). In the absence of humidity and at the reduced gravity, electrostatic forces between these dust particles can dominate its bulk behavior. Any in-situ resource utilization technique will require information on these electrostatic forces.

This study investigates a simple case: the angle of repose of a particle system. This system is governed by the combined effect of particle–particle friction, electrostatic charges, and gravity. For a variety of geotechnical applications the angle of repose provides the most elementary design information. It will be seen how the electrostatic charges affect the angle of repose for an aggregate of particles.

The angle of repose of a granular pile is the result of the avalanching process. As the surface particles fall off the pile, exposure of the previously shielded particles allows them to be UV charged. In this way, particles actively involved in the avalanching process are always charged. Particles that are buried deep in the pile are irrelevant to the dynamics of the surface. Thus for simplicity, the bulk of the study presented here assumes a uniform charge for all particles. As will be explained in the section entitled "Model Description," the simulation calculates the Coulomb force only between immediate neighboring particles, hence the dynamics of the surface particles are reasonably close to the realistic lunar case. The model may be extended in the future to allow only exposed particles to be charged and to treat the Coulomb force more exactly as a long-range force between all the grains in a reasonable distance.

The writers emphasize that this is a conceptual study aimed at finding the qualitative effect of electrostatic forces on bulk behaviors of a particle system. A discrete element method (DEM) is utilized to simulate a two-dimensional (2D) system of disks. Generalization to three dimensions (3D) will require massive computing resources and hence is left for future work.

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## Model Description

The angle of repose is a classic method in soil mechanics to get friction properties of soil near the surface. It is needed for designing stable embankment. It has also been used to analyze avalanches and the mixture of granular materials in industrial drums (Dury et al. 1998; Jaeger et al. 1989). In order to determine whether electrostatic charges can affect the angle of repose, a 2D DEM is used in this study. The simulation strategy and contact force model used are the same as described in Babic et al. (1990).

Because of the irregular shape of lunar dust particles, two particle shapes are studied to see if the writers' findings are robust to shape factor. These two cases are: single disks and clumps formed with a pair of overlapping disks. In both cases, a multiple size distribution of particles with a uniform probability function is used. A range of electrostatic forces and two gravity fields, $g_E = 9.8\ \text{m/s}^2$ and $g_M = 1.62\ \text{m/s}^2$, are studied.

In the simulation, the particles contact with the usual viscoelastic model. In addition, the electrostatic forces are also included. The linear viscoelastic contact force is defined as (Babic et al. 1990)

$$F_c = F_{cn} + F_{cs} \tag{1}$$

in which the normal contact force is

$$F_{cn} = K_n x_n + C_n \dot{x}_n \tag{2}$$

The tangent force is modeled in the same way but constrained by the friction limit

$$F_{cs} = \min(\mu K_n x_n, K_s x_s) \tag{3}$$

In Eqs. (2) and (3), $x_n$ and $x_s$ = normal and tangential displacement and $\dot{x}_n$ = normal deformation rate. The particle stiffness is characterized by the normal stiffness $K_n$ and tangential stiffness $K_s$. The normal stiffness is related to the Young's modulus of the particles: $K_n \sim ED$, where $D$ = particle diameter.

The interaction forces between particles also include the capillary adhesion and electrostatic forces (Starukhina 2005):

$$F_i = F_a + F_e \tag{4}$$

where the capillary adhesion is

$$F_a = 2\pi r_l \Delta\alpha \tag{5}$$

in which $r_l$ = local radius of curvature in the contact zone and $\Delta\alpha$ = result of the molecular bonding of the individual particles when it was formed and the bonding at the interface when two particles are brought into close contact. Its value is a function of the particle composition. The electrostatic force is

$$F_e = \frac{q_a q_b}{4\pi\varepsilon_0 r^2} \tag{6}$$

where $r$ = distance between center of mass of the two particles; $q_a$ and $q_b$ = charges of particle $a$ and particle $b$, respectively; and $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2$ = dielectric constant for vacuum. Lunar dust particles are extremely irregular in shape with asperities at many different scales. Therefore it is expected that the radius of curvature at a contact is very small and hence the attractive force as defined in Eq. (5) is small and ignored in this study.

In the simulation, only close neighbors within a distance of roughly three mean particle diameters are considered for the electrostatic forces. Farther out the electrostatic forces drop off as $1/\text{distance}^2$ and hence become negligible rapidly. Considering only close neighbors is also consistent with the fact that only surface particles are UV charged.

**Table 1.** Particle Parameters Used in the DEM Simulations

| Variable | Definition | Value |
|---|---|---|
| $D$ | Mean particle diameter | 100 μm[a] |
| $D_{max}$ | Maximum diameter | 110 μm |
| $D_{min}$ | Minimum diameter | 90 μm |
| $K_n$ | Normal stiffness | ~1.0E2 N/m |
| $K_s$ | Shear stiffness | ~0.8E2 N/m |
| ρ | Particle density | 3000.0 kg/m[3a] |
| $e$ | Restitution coefficient | 0.5 |
| $g_E$, $g_M$ | Gravity on Earth and Moon | 9.81 m/s² (Earth)<br>1.62 m/s² (Moon) |
| $q$ | Electrostatic charge | 0.0, $10^{-15,-14,-13,\text{or}\ -12}$ C |
| μ | Particle–particle friction | 0.0, 0.1, 0.5, 1.0, 1.5 |
| $\mu_{wb}$ | Particle–floor friction | 0.5 |
| | Total mass simulated | $1.65 \times 10^{-6}$ kg (698 single particles or 349 pairs) |

[a]Heiken et al. 1991

To simulate lunar dust the present 2D model assumes particles are disk in shape with a uniform probability distribution from 90 to 110 μm in diameter and a uniform thickness of $D$ = 100 μm. In this way, although the geometry is 2D, the actual mass and thus the gravitational effect on these particles are very similar to the 3D particles. Two types of particles are used in the angle of repose simulation. One is a single disk and the other is a paired disk. The paired disk consists of two disks randomly chosen from the size distribution. These two disks are allowed to overlap with the radius of the smaller disk. The paired disk is used to better represent nonspherical particles encountered in a real soil. The other parameters are defined in Table 1.

The restitution coefficient is speculated. As this coefficient only dissipates the dynamic energy of the system, it has negligible effect on this quasi-static case of formation of the angle of repose. The stiffness is much lower than the ordinary soil particles. The real stiffness parameter for 100 μm size soil particles is on the order of $10^5$ N/m. Because the computing time increases with $\sqrt{K_n/m}$, where $m$ = particle mass, the real stiffness cannot be achieved at this time. Fortunately, the system studied is under very little load from only the self-weight of the materials. The deformation of particles and the stiffness are therefore negligible. The range of the uniform electrostatic charge $q$ is assumed to be between $10^{-15}$ and $10^{-12}$ C. Direct measurements of lunar dust particles and lunar and Martian simulant provided an average charge on the order of $10^{-14}$ C for 100 μm particles under laboratory conditions (Sickafoose et al. 2001; Sternovsky and Robertson 2002). It was also found that this charge increased linearly with particle diameter (Sternovsky and Robertson 2002). Similar data were also obtained for Ottawa quartz (Green et al. 2003).

All simulations are carried out on IBM single processor computers with 3 GHz processor and 1.0 Gbytes random-access memory. For the angle of repose calculations, each case consists of roughly 700 particles for the single-disk systems and 350 pairs for the overlapping disk pairs. Each simulation requires 12 to 42 h of run time to reach the final static condition. A total of 118 simulations are carried out for this study, of which 108 are for the friction and electrostatic charge investigation, 2 are for the wall and particle interaction investigation, and 8 for mixed positive and negative charge effects.

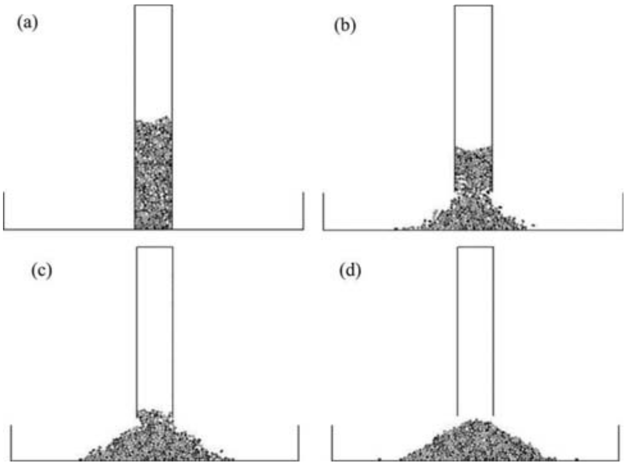


**Fig. 1.** Sequence of the formation process of the angle of repose

## Results

As shown in Fig. 1, the angle of repose is generated with a rectangular bin with the size 97.5*D* (height) by 15*D* (width), which is filled with particles at the start of the simulation. The bottom of this bin sits on the floor of a container. The width of the container is 120*D* with height 15*D*. All particles in the bin are at rest at the beginning of the pile formation. The friction between the particles and the bin is set at 0.5. To initiate the piling up of the particles and minimize the disturbance from the bin, the walls of the bin are "melted" away from the floor to expose the particles from the bottom upward. In the simulation, the "melting" is implemented by removing the walls step by step at each computational time step. The speed of this melting is set at 5 mm/s for the Earth gravity and at 2.5 mm/s for the Moon gravity. The effect of melting speed is found to change the results by less than 5% for particle systems consisting of 1 mm particles. But such effect has not been investigated for the current system.

As the walls are melted, at first particles in the bin fall out and spread over the floor of the containing box. When more particles flow out, a pile begins to form. The surface of the pile performs small scale avalanches to form the final angle of repose. After all particles are discharged, the pile takes a few more moments to complete the avalanche and arrive at the static final shape. A sequence of this formation is shown in Fig. 1.

Due to the discrete nature of the system and the relatively low number of particles, any visual fitting of the surface slope to obtain the angle or repose is subjective. To define an objective measure for the angle of repose, this study takes the static final pile, filters out all interior particles first, then uses a least-squares fit of the surface particles to define the slopes of the two sides of the pile. The angle of repose for the pile is the average angle from the two slopes. It is worth mentioning that the two vertical walls of the container are necessary to establish the pile for the circular particle case. Without them, circular particles can easily roll out. Consequently, the base of the pile horizontally expands indefinitely. This phenomenon is much less significant for clumped particles or for circular particles with sufficient rolling friction.

Three independent simulations are carried out for each case. Each simulation used a different random number sequence to form a different packing inside the bin. The resulting angle of repose presented in the following includes the mean and the standard deviation of the three simulations.

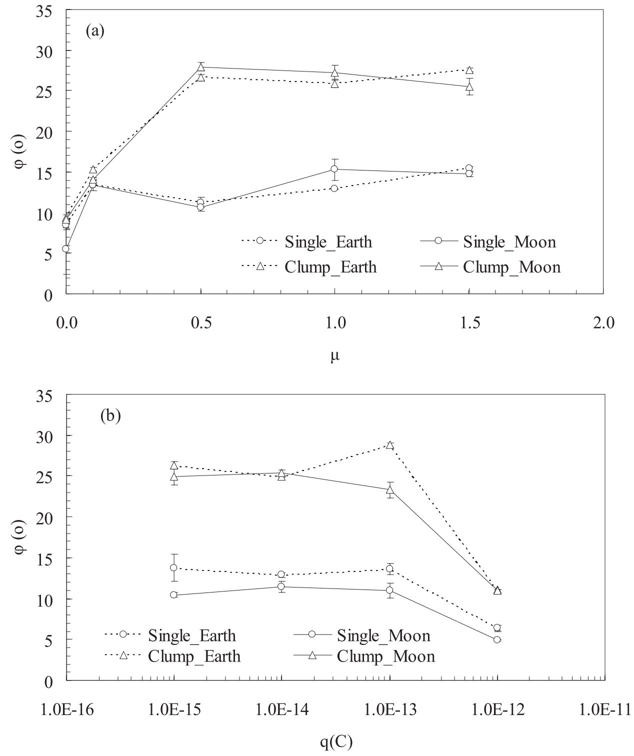


**Fig. 2.** Angles of repose with respect to contact friction and electrostatic charge with wall separation equal to 120*D*.

Fig. 2 summarizes the results of the simulations. In Fig. 2(a), the electrostatic charge is zero, and in Fig. 2(b), the particle friction coefficient is fixed at 0.5. It is observed that:

1. The angle of repose of single-disk particles is much smaller than that of paired-disk particles when the contact friction is greater than 0.5.
2. For all cases of friction and electric charges studied the angle of repose in both gravity fields is nearly identical.
3. The angle of repose increases with the particle surface friction coefficient for both single-disk and paired-disk particles. However, such dependence saturates at high contact friction. Further increase of contact friction has negligible effect on the angle of repose.
4. Increasing electrostatic charges decreases the angle of repose. However, at very weak electrostatic charges even a twofold increase in charges has almost no effect on the angle of repose. When a threshold is reached, the angle of repose drops sharply with small increases of the electrostatic charges.
5. The standard deviation of all cases is small, therefore the dependence on the initial condition appears to be insignificant.
6. The above-mentioned results are from the cases where friction between the wall of the bin and the particles is 0.5. Two cases—one with single particles and one with clumped particles—were tested where the friction between the particles and the wall of the bin is 0. It was found that the angle of repose differed from the 0.5 case by 3°. Hence, the effect of the wall is similar to the statistical variation within each case.

## Discussion

The angle of repose is nearly independent of the gravity when particles are not charged. This result should be expected. Without charge, the angle of repose is determined when the frictional force of the surface material is equal to the weight component in the slope direction. The tangent of this angle is the ratio of the gravity component in the slope direction to the normal direction. Therefore, the magnitude of gravity is irrelevant. The interparticle force is the only determining factor. However, when electrostatic forces are present, the repulsive force works against gravity to destabilize the pile. This destabilizing force does not scale with gravity, hence its effect becomes significant when gravity is low. It is thus expected that the angle of repose will differ between the Earth case and the Moon case when electrostatic forces are considered. From Fig. 2(b) such effect is not obvious in the cases studied. A set of simulations not included here were carried out for 1 mm diameter and 1 m long rod shape particles where it was found that with electrostatic charges the angle of repose did reduce under the Moon gravity.

The angle of repose is sensitive to the contact friction below a certain value (0.5 for the cases studied here) and to electrostatic charge above a certain value ($10^{-13}$ C for the cases studied here). This threshold value is well below the electron field emission value estimated at $5.56 \times 10^{-10}$ C for 100 $\mu$m size particles. The ion field emission value is even higher.

The exact values of such threshold phenomena are functions of the particle properties such as the particle shape. To design tools and equipment for handling lunar surface materials the friction and electrostatic charges of this material are important parameters. It is desirable to have a simple and reliable method to quantify these parameters. The sensitivity of the angle of repose to the friction coefficient and to the electrostatic charges outside the threshold may provide ways to determine these parameters without measuring them from individual particles.

The reliability of the above-presented observation needs to be further investigated. All results are from a small sample size and with only three independent initial packings. The angle of repose is a statistical variable. The variability of its value can only be reduced if the particle number and the number of independent tests are large. Limited by the computing time requirement, such study awaits future work.

To simulate a real lunar soil, a great deal of improvement over the present model is necessary. For instance: apply charges to a few surface layers of particles and leave the bulk interior charge free, calculate the Coulomb force over a longer range, extend to 3D cases, include a broader size distribution, and allow a distribution of charges. Of the improvement to be made, the writers expect that the extension to three dimensions will produce statistically more consistent results for the angle of repose, because the measurement of the pile angle can be averaged over 360° instead of only two sides.

Finally, the above-presented study assumes all particles are unanimously positively charged or negatively charged. Although such assumption is consistent with the focus on photoemission-induced charges, triboelectrification is excluded. Triboelectrification is a common process in sheared particle systems. Such phenomenon will be extremely important for any in-situ resource utilization operations. Based on the findings here, one might wonder how a mixed charge such as resulted from the triboelectric process would affect the angle of repose. The footprint left by the astronauts on the Moon during the Apollo 11 mission showed a nearly vertical angle of repose. Could it be the result of triboelectric charge? To test this idea, a system that is identical to the above-presented cases is simulated, except that the charges on those particles have a normal distribution between $\pm 5 \times 10^{-14}$ C and between $\pm 1 \times 10^{-13}$ C with a zero mean. The resulting angles of repose are found to be about the same as the no charge case. The lack of influence due to mixed charge is because the grouping of particles from opposite charges is similar to having a system of larger but neutral particles.

## Conclusion

Using DEM simulations the behavior of the angle of repose of particles with electrostatic charges under the Earth gravity and the Moon gravity was investigated. It has been found that the charges on the particles have a threshold value. For charges below this value the angle of repose is independent of the charge, and above this value the angle of repose is very sensitive to the charges. The existence of this threshold may provide an indirect way to measure the electrostatic forces between particles that are charged by photoemission processes. The work presented here should be viewed as a conceptual study as it is two dimensional, with uniform charge on particles, and with a particle size distribution and particle shape very different from the real lunar dust.

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## Notation

*The following symbols are used in this paper:*

$C_n$ = damping coefficient;
$D$ = mean particle diameter;
$D_{\max}, D_{\min}$ = maximum and minimum diameter;
$e$ = restitution coefficient;
$F_c, F_{cn}, F_{cs}$ = contact force, its normal and shear components, respectively;
$F_i, F_a, F_e$ = surface interaction force, capillary attractive and electrostatic force, respectively;
$g_E, g_M$ = gravity on Earth and Moon;
$K_n$ = normal stiffness;
$K_s$ = shear stiffness;
$Q, Q^*$ = mass flow rate, dimensionless mass flow rate, respectively;
$q$ = electrostatic charge;
$r$ = distance between centers of two particles;
$x, \dot{x}$ = particle overlap, particle relative velocity, respectively;
$\alpha_1, \alpha_2$ = specific surface energy;
$\varepsilon_0$ = dielectric constant;
$\mu$ = particle friction;
$\mu_{wb}$ = particle-floor friction; and
$\rho$ = particle density.

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